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Introducing the dilatational degree of freedom: special relativity in V_6

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Abstract. We argue that the assumption of the universal atomic scale is not justified by observations. The latter indicate just the contrary, namely that the scale is a *degree of* freedom. We attempt to state a concise general definition of *the principle of relativity*. The incorporation of the dilatational degree of freedom into the principle of relativity requires that physical events take place in the six-dimensional space V_6, the subspace of which is the Minkowski space M_4. The norms and angles of 6-vectors in V_6 are assumed to be conserved, whilst their projections into M_4 are in general not conserved under the transformations of the group ISO(4, 2). The speed of light is invariant in V_6 but its projection into M_4 is invariant only under certain transformations, for instance under those of the Poincaré group ISO(3, 1) $\subset$ ISO(4, 2). The theory provides a description of the muon to electron mass ratio. The 'primordial' transverse momentum of partons is proposed to originate from *the dilatational momentum*.

1. Introduction

Special and general relativity in space–time encompasses a wide area of observed phenomena, ranging from astrophysics to the physics of subnuclear particles. In spite of this success there are several problems. Relativity in space–time, apart from the problem of its quantisation, is far from being able to represent in a theoretically satisfactory way all presently observed phenomena. In particular, it does not unify different 'fundamental' interactions. There are also several unresolved problems in astrophysics, such as the controversial anomalous red shifts of galaxies (Arp 1970, Field *et al* 1973, and references therein), and the very large 'velocity' dispersion in clusters of galaxies (Clube 1978). In the field of subnuclear particles there are the problems with the so-called large transverse momentum phenomena (Jacob and Landshoff 1978). Moreover, the origin of the e/μ mass ratio is not known.

In the situation when new experimental data are being accumulated, not easily fitted into the existing relativity and quantum electrodynamics or quantum chromodynamics, we have to become more open in considering new ideas concerning the very fundamentals of the theories. In the present paper I propose a generalisation of relativity which incorporates the hypothetical *dilatational degree of freedom* associated with the assumed *variability of the scale of an object.* Let the length of a 4-vector connecting two different events E_1, E_2 within an object be a measure of an object's *scale.* If an object serves as a clock let the events E_1, E_2 be two successive 'ticks'. If an object serves as a rod let the events E_1, E_2 be emissions of light signals from the ends of the rod. I assume that in a given reference frame in the four-dimensional space–time an object's scale is variable along its world line (for an extended object: world tube).

Before explaining what exactly I mean by the 'variable scale' of an object let me say some words about the concept of scale.

I wish to demonstrate that the concept of scale has not been fully examined so far. More open and more rigorous considerations of this concept will result in a theory which will provide new interpretational possibilities for some observed—but not well understood—phenomena.

In the excellent book by Dicke (1964) it is written: 'Imagine, if you will, that you are told by a space traveller that a hydrogen atom of Sirius has the same diameter as one on the earth. A few moments' thought will convince you that the statement is either a definition or else meaningless. It is evident that two rods side by side, stationary with respect to each other, can be intercompared and equality established in the sense of an approximate congruence between them. However, this cannot be done for perpendicular rods, for rods moving relatively, or for rods with either a space- or time-like separation. Their intercomparison for purposes of establishing equality cannot be made until rules of correspondence are established'.

Let O_0 be an object—called the *reference object*—defining a *reference length* $d(O_0)$. The quantity $d(O_0)$ is defined to be a 4-distance between two chosen events E_1 and E_2 related to the object O_0. If O_0 serves as a *clock*, then let E_1 and E_2 be two successive 'ticks' separated by a time-like 4-distance. If O_0 serves as a measuring rod, then let E_1 and E_2 be chosen to be separated by a space-like 4-distance. In the case when O_0 is a *hydrogen atom* let the 4-distance $d(O_0)$ be chosen to represent in a rest frame:

(i) when being time-like, the time interval between two successive minima of the emitted light wave;

(ii) when being space-like, the diameter of O_0.

Let O be an arbitrary object, its length $d(O)$ being defined in the same way as $d(O_0)$. The scale κ of O relative to O_0 is defined by

$$\kappa = d(O_0)/d(O). \tag{1.1}$$

In equation (1.1) the scale κ_0 of the reference object O_0 has been taken to be $\kappa_0 = 1$ by definition.

The simplest way of measuring the scale of O is to put both O and O_0 side by side and to compare their lengths. To compare their lengths means either to compare their diameters or their characteristic time intervals.

However, in many cases we wish to compare the length of a *distant* object O with the length of a reference object O_0. In such a case, in order to obtain the diameter $d(O)$ of O, we must determine both the angular diameter of O and the distance $\overline{OO_0}$. The distance $\overline{OO_0}$ is defined in terms of the time that it takes light to pass from O_0 to O and back to O_0 (Anderson 1967).

In his last scientific writing Einstein (1955) realised that in principle there should exist objects O_1, O_2 identical in all respects except for having different scales κ_1, κ_2. When put *side by side* such objects would have *different lengths* $d(O_1)$, $d(O_2)$, their ratio being

$$\frac{d(O_1)}{d(O_2)} = \frac{\kappa_2}{\kappa_1}. \tag{1.2}$$

When O_1 and O_2 are observed from the distances l_1 and l_2, respectively, such that $l_1/l_2 = \kappa_1/\kappa_2$, then O_1 and O_2 would have identical angular structures (and therefore equal angular diameters). Einstein rejected this possibility, since it seemed to him to be contrary to the observed facts. That is to say, according to him and to almost everybody, the observed fact is that all hydrogen atoms in the universe have equal scales which implies equal diameters and equal frequencies of equivalent natural emission lines. My opinion is that this is not an observed fact. *The possibility of similar objects with different scales is not discredited by observations*, as I am going to show. As given above, in order to know the scale of an object we must know its length. In a rest frame, the length may be either *diameter* or characteristic *time interval.*

For a given observer *the diameter* $d(O)$ of a distant object O is uncertain if the distance between observer and O is uncertain. Therefore, the scale $\kappa = d(O_0)/d(O)$ is also uncertain to the same extent. It is a well-known fact in astrophysics that the distances of distant objects are not known very precisely. *A consequence is then that scales of these objects (stars, galaxies, etc) are also uncertain. Therefore, the possibility that different identical objects—for instance, hydrogen atoms—in the universe might have different scales is not excluded.* If such hydrogen atoms with different scales would be translated in space–time so as finally to be placed side by side, their scales would still be different. In fact, by definition, *translation* in space–time is such a transformation that changes an object's position but leaves its scale unchanged (see Pavšič 1977 and references therein). On the other hand, *dilatation* changes an object's scale but leaves its position unchanged. So an object has both *translational* and *dilatational* degrees of freedom. *If an object is free, i.e. has no forces acting on it, its position and/or scale varies uniformly or is constant along its world line*, as observed from a given reference frame defined by world lines of reference objects. We will say that an object is *moving translationally* and/or *dilatationally. Thus the concept of inertia is also extended from translational degrees of freedom to the dilatational degree of freedom.* This will be more extensively explained in the subsequent sections.

An appropriate measure of *time interval* associated with a hydrogen atom is the frequency ν of a typical emission line. Let this hydrogen atom belong to a macroscopic object O. Let ν_0 be the frequency of the corresponding equivalent emission line from a reference hydrogen atom belonging to a macroscopic object O_0. Let us require for a moment that O is translationally at rest with respect to O_0. *The scale* κ of the hydrogen atom in O is defined by

$$\kappa = \nu/\nu_0 \tag{1.3}$$

where the scale κ_0 of the reference hydrogen atom is $\kappa_0 = 1$. I must stress that in equation (1.3) both ν and ν_0 refer to equivalent emission lines, e.g. K_α lines.

If O moves translationally with a non-relativistic velocity $v \ll c$ towards O_0, the observed frequency is

$$\nu = \kappa\nu_0(1 + v/c). \tag{1.4}$$

The frequency shift is

$$\Delta\nu/\nu_0 = \kappa v/c + \kappa - 1. \tag{1.5}$$

Only in the special case $\kappa = 1$ do we obtain the familiar red shift or blue shift relation $\Delta\nu/\nu_0 = v/c$. Equation (1.5) teaches us that *for the measured shift* $\Delta\nu/\nu_0$ *the scale* κ *is uncertain to the same extent that the velocity* v *is uncertain.* If O is a distant star or a galaxy, its velocity v is uncertain. So we do not know the scale of a hydrogen atom in a star or a galaxy. This uncertainty in scale is reduced but not eliminated by additional information such as the knowledge of the angular diameter of O and its approximate distance from the Earth. So it is possible that *different identical objects in the universe might have different scales.*

Let me now say some words about identical objects. A solar system and a galaxy have different scales. However, these objects are not identical, since a solar system cannot be magnified to become congruent with a galaxy. In other words, a galaxy and a solar system cannot be seen from different distances so as to appear as objects with identical observed angular structures (and angular diameters). In the present paper I assume and explore the possibility (Pavšič 1975, 1977) of the existence of *identical objects having different scales* (Caldirola *et al* 1978a, b), for instance hydrogen atoms having different scales and therefore different frequencies of equivalent emission and absorption lines. The statement: 'all hydrogen atoms in the universe have equal scales' is *an assumption* not confirmed by observations. If so, we may as well assume the contrary, namely that different hydrogen atoms have in general different scales, and then find out the consequences of a theory based on such an assumption.

Let us postulate the following. *The scale of 'fundamental' physical quantities such as the Bohr radius, charge, mass of electron, etc, is not absolute but relative to a given reference frame.*

A reference frame—composed, for instance, of reference hydrogen atoms—may change so that we obtain different values for those quantities. In a given reference frame it is possible to find atoms, nuclei, etc, having scales that differ from the scales of presently known atoms, nuclei, etc. By postulating this, questions like 'why is the mass of an electron 0·51 MeV and not 5 MeV, the charge $1{\cdot}6\times10^{-19}$ C and not 7×10^{-18} C?' are meaningless. Only questions like 'why is the ratio of the mass of the proton to the mass of the electron in the bound system such as a hydrogen atom 1937?' have meaning. When an atom is broken into a free electron and a free proton, their mass ratio remains the same as it was within their bound system except for the bound energy corrections. Protons and electrons ejected from various atoms have, in principle, various mass ratios, provided that the scales of atoms are different. However, atoms forming or being ejected from a certain system, such as a crystal or a sample of air, in which they experience mutual interactions, have equal scales within the present accuracy of measurements. Possible differences in their scales could be detected even in a terrestrial laboratory as an additional spread added to the natural width, the Doppler spread, etc, of a spectral line of a sample material (cf equation (1.4)).

According to the position assumed in the present paper, *scale is a degree of freedom, called the dilatational degree of freedom*, possessed by an object besides the translational and rotational degrees of freedom. In order to include scale in a theoretical representation of the world I propose that, instead of the Minkowski space M_4, we have to use the six-dimensional non-compact space V_6. A physical event is represented by six *independent* coordinates (Kastrup 1962, Barut and Haugen 1972) $\eta^a\equiv(\eta^\mu,\eta^5,\eta^6)$ ($a=0,1,2,3,5,6$; $\mu=0,1,2,3$) with $\eta^\mu=\kappa x^\mu$, $\eta^5\equiv\kappa$, $\eta^6\equiv\lambda$. When projected into the Minkowski subspace $M_4\subset V_6$, the coordinates η^μ, κ, λ are related, respectively, to the changes of position, changes of a 4-vector length, $\mathrm{d}s=(\mathrm{d}x^\mu\,\mathrm{d}x_\mu)^{1/2}$, and the changes of the angles $\cos\varphi=\mathrm{d}x_1^\mu\,\mathrm{d}x_{2\mu}/(\mathrm{d}s_1\mathrm{d}s_2)$. In the presence of the algebraic constraint $\eta^a\eta_a=\eta^\mu\eta_\mu-\kappa\lambda=0$ (i.e. $\lambda=\kappa x^\mu x_\mu$), $\cos\varphi$ is an invariant quantity and we deal with *the conformal relativity in the restricted sense* (Cunningham 1909, Bateman 1910, Kastrup 1962, Barut and Haugen 1972). In the present paper I do not impose any constraint on the coordinates η^a, and therefore $\cos\varphi$ is not invariant under *all* linear orthonormal transformations which preserve the quadratic form $\mathrm{d}\eta^a\,\mathrm{d}\eta_a=$

$\mathrm{d}\eta^{\mu}\,\mathrm{d}\eta_{\mu}-\mathrm{d}\kappa\,\mathrm{d}\lambda$ and the angle between the 6-vectors $\mathrm{d}\eta_1{}^a, \mathrm{d}\eta_2{}^a$. These transformations belong to the inhomogeneous orthonormal group ISO(4, 2) of rotations and translations in the space V_6.

2. Definition of the principle of relativity

We will take an observation as a primitive element of a theory. An *observation o* is an event or set of events which happen in a measuring instrument R. An instrument R can register many—in practice infinitely many—various observations which reflect the physical situations the surrounding world happens to find itself in. An observation o registered by R changes either when the surrounding world changes or when R changes. Let $\{o\}$ be the set of all possible observations $o \in \{o\}$ which can be registered by R, and let $\{R\}$ be the set of all possible instruments $R \in \{R\}$ which are available, at least in principle, to an observer.

Let $\mathscr{A}$ be a group of the transformations $A \in \mathscr{A}$ such that each A changes an instrument R into another instrument R':

$$R = AR' \qquad R' = A^{-1}R. \tag{2.1}$$

The group $\mathscr{A}$ defines a subset $\{S\} \subset \{R\}$ of instruments S, S', etc, such that for each $S \in \{S\}$ and for each $S' \in \{S\}$ we have

$$S = AS' \qquad S' = A^{-1}S. \tag{2.2}$$

When passing from S to S' an observation o registered by S transforms to an observation o' registered by S':

$$o' = Ao. \tag{2.3}$$

The set $\{R\}$ is the union of all possible subsets $\{S\}$. Let $\{S\}$ be called the *equivalence class of instruments.*

With the aid of the group $\mathscr{A}$ it is also possible to define a subset $\{o_e\} \subset \{o\}$ of observations o_e, o'_e etc, such that for each $o_e \in \{o_e\}$ and for each $o'_e \in \{o_e\}$ we have

$$o'_e = Ao_e \qquad o_e = A^{-1}o'_e. \tag{2.4}$$

Let a subset $\{o_e\}$ be called the *equivalence class of observations.* The set $\{o\}$ of all possible observations is the union of all various equivalence classes $\{o_{e_1}\}, \{o_{e_2}\}, \ldots$:

$$\{o\} = \bigcup_i \{o_{e_i}\} \tag{2.5}$$

(i runs over all various equivalence classes).

Let a relation among various observations $o_1, o_2, \ldots,$ be called a *physical law* (Recami and Mignani 1974).

Let us introduce *the principle of relativity* defined by the following requirements.

(a) For an arbitrary $S \in \{S\}$ all various equivalence classes of observations are isomorphic to each other.

(b) Each equivalence class $\{S\}$ of instruments is isomorphic to each equivalence class $\{o_e\}$ of observations.

(c) A physical law is covariant, i.e. invariant in form under any $A \in \mathscr{A}$.

If the requirements (a), (b), (c) of the principle of relativity are satisfied we will say that S is a *reference frame* and that $\mathscr{A}$ is *the covariance group* of a theory about the observations o.

Let the totality of all *frame-independent* concepts that we can derive from observation be called the *phenomenon O*. An observation is related to a corresponding phenomenon O:

$$o = O(S). \tag{2.6}$$

A phenomenon is invariant but an observation depends on the reference frame S. For special relativity in the Minkowski space M_4 a phenomenon is, for instance, the world line (or world tube) of an object, whilst an observation is its velocity.

The goal of an experiment is to enlarge $\{o\}$, whilst the task of a theory is to find out a corresponding *principle of relativity*, based on the requirements (a), (b), (c). If the principle of relativity cannot be determined, the theory artificially completes the set $\{o\}$ of the existing observations so as to be able to satisfy the principle of relativity, and thus *predicts* new possible observations. On the contrary, new experimental discoveries can enlarge the set of existing observations, so that they are no longer encompassed in the old principle of relativity which is based on the old covariance group. In such a case, the covariance group must be enlarged so as to encompass the new observations also.

In the present paper I start from the point of view that the existing observations, especially those in astrophysics, no longer permit the Poincaré group ISO(3, 1) as a covariance group. Instead of ISO(3, 1) we have to take at least the group ISO(4, 2) as a covariance group.

3. Mathematical preliminaries: definition of the group ISO(4, 2)

Let V_6 be a six-dimensional non-compact space with a metric tensor γ_{ab} of signature $(+----+)$. Let η^a be the contravariant and η_a the covariant components of a position 6-vector in the space V_6. Let the inhomogeneous group of transformations L_I that preserve the quadratic form

$$\mathrm{d}\sigma^2 = \mathrm{d}\eta^a\,\mathrm{d}\eta_a = \gamma_{ab}\,\mathrm{d}\eta^a\,\mathrm{d}\eta^b \qquad (a, b = 0, 1, 2, 3, 5, 6) \tag{3.1}$$

be denoted by ISO(4, 2). Its homogeneous subgroup is just the well-known conformal group SO(4, 2). This is the reason for the choice of signature $(+----+)$.

For physical reasons it is more convenient to use a basis in which the metric tensor is non-diagonal (Kastrup 1962, Barut and Haugen 1972):

$$\delta_{ab} = \begin{pmatrix} \delta_{\mu\nu} & 0 & 0 \\ 0 & 0 & +\frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{pmatrix} \qquad (\mu, \nu = 0, 1, 2, 3) \tag{3.2}$$

where $\delta_{\mu\nu} = \mathrm{diag}\,(+---)$. The contravariant components of a position 6-vector are then

$$\eta^a \equiv (\eta^\mu, \kappa, \lambda) \qquad (\kappa \equiv \eta^5, \lambda \equiv \eta^6) \tag{3.3}$$

whilst the covariant components $\eta_a = \delta_{ab}\eta^b$ are

$$\eta_a = (\eta_\mu, -\tfrac{1}{2}\lambda, -\tfrac{1}{2}\kappa) \qquad (\eta_\mu = \delta_{\mu\nu}\eta^\nu). \tag{3.4}$$

In a basis in which γ_{ab} is given by equation (3.2) the quadratic form (3.1) is

$$d\sigma^2 \equiv d\eta^\mu \, d\eta_\mu - d\kappa \, d\lambda. \tag{3.1a}$$

Instead of the coordinates η^a we can use as well the coordinates $x^a \equiv (x^\mu, \alpha, \Lambda)$:

$$x^\mu = \kappa^{-1}\eta^\mu \tag{3.5a}$$

$$\alpha = \kappa^{-1} \tag{3.5b}$$

$$\Lambda = \lambda - \kappa^{-1}\eta^\mu\eta_\mu. \tag{3.5c}$$

In the coordinates x^a the quadratic form (3.1a) becomes

$$d\sigma^2 = \alpha^{-2}(dx^\mu \, dx_\mu + d\alpha \, d\Lambda) = \alpha^{-2} \, d\tilde{s}^2 \tag{3.6}$$

where $d\tilde{s}^2 \equiv dx^\mu \, dx_\mu + d\alpha \, d\Lambda$. Under the action of a transformation $L_I \in \mathrm{ISO}(4, 2)$

$$d\sigma'^2 = d\sigma^2 \qquad \text{i.e. } \alpha'^{-2} \, d\tilde{s}'^2 = \alpha^{-2} \, d\tilde{s}^2 \tag{3.7}$$

where in general $\alpha' \neq \alpha$.

The theory is simpler in terms of the coordinates η^a, since the quadratic form $d\sigma^2$ is invariant under $L_I \in \mathrm{ISO}(4, 2)$. On the other hand, results of observations are more obvious in terms of the coordinates x^a, since the first four components x^μ by definition represent the usual space–time coordinates. Further, from (3.5) and (3.7) it follows that the coordinate α determines the scale of $d\tilde{s}$ relative to a fixed $d\sigma$. In general $d\tilde{s}^2$ changes when passing from a reference frame S in which the scale is α to another frame S' in which the scale is $\alpha' \neq \alpha$ (cf equation (3.7)). The coordinate Λ manifests itself through a deviation of 4-angle cosines:

$$\cos\varphi = \frac{dx_1^\mu \, dx_{2\mu}}{ds_1 \, ds_2} = (\alpha^2 \, d\eta_1{}^a \, d\eta_{2a} - \tfrac{1}{2} \, d\alpha_1 \, d\Lambda_2 - \tfrac{1}{2} d\alpha_2 \, d\Lambda_1)(ds_1 \, ds_2)^{-1} \tag{3.8}$$

where $ds_1 = (dx_1^\mu \, dx_{1\mu})^{1/2} = (\alpha^2 \, d\eta_1{}^a \, d\eta_{1a} - d\alpha_1 d\Lambda_1)^{1/2}$ and analogously for the index 2.

We have two possibilities:

(i) $\Lambda \neq 0$; then $\cos\varphi$ is invariant only with respect to those $L_I \in \mathrm{ISO}(4, 2)$ that preserve $d\alpha$ and/or $d\Lambda$;

(ii) $\Lambda = 0$; then it is (cf equation (3.5))

$$\eta^a\eta_a = \eta^\mu\eta_\mu - \kappa\lambda = 0 \tag{3.9}$$

which is invariant under the homogeneous transformations $L \in \mathrm{SO}(4, 2)$; in this case for $\Lambda = 0$, therefore, $\cos\varphi$ also is invariant under $L \in \mathrm{SO}(4, 2)$, as follows from equation (3.8).

The imposition of the condition

$$\Lambda = 0 \Leftrightarrow \eta^a\eta_a = 0 \Leftrightarrow \lambda = \kappa x^\mu x_\mu \tag{3.10}$$

implies the transition from relativity in the space V_6 to relativity in the space $V_5 \subset V_6$. The space V_5 is the five-dimensional hypercone, defined by equation (3.10), embedded in V_6. V_5 is usually called *conformal space* (Barut and Haugen 1972) and the relativity in this space *conformal relativity* (Ingraham 1978). Even if the latter is called 'relativity' it does not satisfy the principle of relativity as defined in § 2, since there is no equivalence set of observations $\{o_e\}$ isomorphic to the equivalence set of reference frames $\{S\}$. Moreover, in the case of a constraint on η^a which results in $\Lambda \neq 0$ we no longer have a six-dimensional *flat* space but a five-dimensional curved space with constant curvature. This curvature does not result from a distribution of matter but, on the contrary, is given *a priori*. This is contradictory to the principle of relativity, and also to the requirement that the space (in this case the five-dimensional space V_5) should be asymptotically flat in the absence of matter.

The principle of relativity can be satisfied only if we abandon the condition $\Lambda = 0$ (*equation* (*3.10*)) *and if we postulate the existence of identical objects not only with different 4-positions* x^μ *but also with different scales* α *and different* Λ. In other words, we require that there exist equivalent observations o_e associated with each other by the transformations $L_I \in \mathrm{ISO}(4, 2)$. Such equivalent observations would be, for instance, those of hydrogen atoms with different scales, as already mentioned in § 1. A more complete treatment of equivalent observations related to the group ISO(4, 2) will be performed in § 5.

Being armed with this preliminary formalism we can now turn to physics and apply the principle of relativity to the case of the covariance group ISO(4, 2).

4. Definition of the measuring instrument for x^a

The equation of a five-dimensional hypercone in the space V_6 with the origin at the point η_i^a is

$$(\eta^a - \eta_i^a)(\eta_a - \eta_{ia}) = (\eta^\mu - \eta_i^\mu)(\eta_\mu - \eta_{i\mu}) - (\kappa - \kappa_i)(\lambda - \lambda_i) = 0. \tag{4.1}$$

This is just the six-dimensional analogue of the light cone in the four-dimensional Minkowski space M_4. If we start, as we do, from requiring that the space V_6 instead of M_4 should be used in physics, then it is also natural to require that in general a light signal travels along a path which satisfies equation (4.1). For such a generalised light signal we could invent some other name, but I think it is superfluous.

Instead of the coordinates η^a we can use the coordinates x^a, related to the coordinates η^a by equation (3.5). In terms of the coordinates x^a the above equation (4.1) reads

$$(x^a - x_i^a)(x_a - x_{ia}) = (x^\mu - x_i^\mu)(x_\mu - x_{i\mu}) + (\alpha - \alpha_i)(\Lambda - \Lambda_i) = 0. \tag{4.1a}$$

The index i refers to an initial event.

Equation (4.1a) implies that the speed of light is unity and invariant only in terms of the six coordinates x^a. It is unity (in the units $c = 1$) and invariant in terms of x^μ ($\mu = 0, 1, 2, 3$) only in the special cases of $\alpha = \alpha_i$ and/or $\Lambda = \Lambda_i$.

The coordinates x^a of *an event P* can be determined by means of a measuring instrument S which consists of a clock M and five emitter–absorbers A, B, C, D, E which are all at rest relative to each other. Let us *ascribe* to these absorber–emitters for the indices $a = 1, 2, 3, 5, 6$ the coordinates $x_A^a, x_B^a, x_C^a, x_D^a, x_E^a$. This is arbitrary and serves as a *chosen standard*. Let x_i^a in equation (4.1a) stand for x_A^a, x_B^a, etc. Then (4.1a) represents five equations for five unknowns x^r ($r = 1, 2, 3$), $\alpha \equiv x^5$, $\Lambda \equiv x^6$. Let the clock M send a light signal at the time t_M^- from M to P. At P let the signal be reflected towards M, A, B, C, D, E whose world lines it crosses at the times $t_M^+, t_A, t_B, t_C, t_D, t_E$, respectively. Then the time coordinate $t = x^0$ of the event P is $t = \frac{1}{2}(t_M^+ + t_M^-)$. The other five coordinates x^r, x^5, x^6 can be determined from the five equations (4.1a) for $i = A, B, C, D, E$.

In order to measure α and Λ we must require the emitter absorbers A, B, C, D, E to have, in general, not only different x^r positions ($r = 1, 2, 3$) but also different α and Λ positions ($\alpha_A \neq \alpha_B \neq \alpha_C \neq \alpha_D \neq \alpha_E$, $\Lambda_A \neq \Lambda_B \neq \Lambda_C \neq \Lambda_C \neq \Lambda_D \neq \Lambda_E$). In practice it would be enough to require

$$\begin{aligned} &x_A^r \neq x_B^r \neq x_C^r && x_D^r = x_E^r = x_M^r \\ &\alpha_M = \alpha_A = \alpha_B = \alpha_C && \alpha_D \neq \alpha_E \neq \alpha_M \\ &\Lambda_M = \Lambda_A = \Lambda_B = \Lambda_C && \Lambda_D \neq \Lambda_E \neq \Lambda_M. \end{aligned} \tag{4.2}$$

If relativity in V_6 is indeed correct then there must exist objects satisfying equations (4.2) to give five *independent* equations (4.1a) for $i = A, B, C, D, E$ from which five independent components x^a (for $a = 1, 2, 3, 5, 6$) can be evaluated ($x^0 = t$ is determined from $t = \frac{1}{2}(t_M^+ + t_M^-)$). The objects M, A, B, C, D, E constitute a *measuring instrument S* which represents a *reference frame* for the coordinates x^a (or η^a).

The existence of a clock M and objects A, B, C, D, E which satisfy five independent equations (4.1a) for i = A, B, C, D, E would be a verification of the theory. At the same time it would give us the definition of the measuring instrument for the coordinates. This is in agreement with the requirement that a theory itself must provide the means of measuring the physical quantities it uses.

Identical objects with different α have different sizes; identical objects with different Λ have different observed x^μ structures (different forms). With Λ increasing or decreasing from the value $\Lambda = 0$ an object's x^μ structure is systematically shifted from its 'normal structure' defined for $\Lambda = 0$.

Though the forms and lengths of objects in terms of the coordinates x^μ ($\mu = 0, 1, 2, 3$) in general are not conserved, they are conserved in terms of the coordinates η^a. The forms remain *similar* in terms of the coordinates x^a ($a = 0, 1, 2, 3, 5, 6$), apart from a change of scale α (cf equation (3.7)).

So we have succeeded in conforming to Einstein's objection to Weyl's theory (1918), namely 'if lengths are not conserved why then are forms still conserved?'. The present theory is more general than Weyl's in the sense that it admits not only non-conservation of lengths but also of forms. This implies that, in the presence of an interaction, the lengths and forms of objects would, in general, depend on their histories. But such a dependence holds only for unbound objects and not for bound objects.

A spatial position x^r ($r = 1, 2, 3$), scale α and Λ of an unbound object are not localised. On the contrary, a spatial position, scale and Λ of a bound object are localised. For instance, in the case of a crystal, atoms have fixed discrete spatial positions, apart from the thermal oscillations around their fixed equilibrium. They have apparently also fixed scales and therefore fixed discrete spectra of emission and absorption lines. In the case of a gas, interactions among atoms (or better, among molecules) are much weaker than in a crystal, and therefore their spatial positions and translational velocities are spread. By analogy we also expect that scales and dilatational speeds of atoms should be spread around an average scale and an average dilatational speed determined by the container of the gas. The bigger the container, the bigger is the spread of positions and scales (and Λ) of atoms within a gas. A consequence of a spread of scales and dilatational speeds is a spread of the emission and absorption spectra, as already discussed in the Introduction (see equation (1.4)).

To sum up, an interaction has a double role: it can either cause a separation of positions x^r, scales and Λ of interacting objects, or cause a formation of bound systems.

In the latter case, not only positions x^r of objects within a bound system but also scales and Λ are expected to be localised. So there is also an answer, at this stage only at the heuristic level, to Einstein's main objection to Weyl's theory: 'we do not observe scale (or length) changes of objects'. The answer is that these observations refer to *bound objects*, in which case, according to the above heuristic considerations, we do not expect significant scale changes at all. On the contrary, unbound objects, in general, are rigorously predicted to change their scales relative to each other. Several astrophysical observations indicate that this is indeed true (see §§ 1 and 6).

5. The principle of relativity based on the covariance group ISO(4, 2)

Let us extend the principle of relativity from the Poincaré group ISO(3, 1) to the group ISO(4, 2). The latter group, being the larger one, encompasses a wider set of possible phenomena than does the group ISO(3, 1). Since $\mathrm{ISO}(3, 1) \subset \mathrm{ISO}(4, 2)$, all observations of ISO(3, 1) also belong to observations of ISO(4, 2).

We will apply the requirements (a), (b), (c) of the principle of relativity to our case of the group ISO(4, 2).

First, I will further clarify some concepts of § 2.

Phenomenon is a frame-independent concept. A phenomenon O is, for instance, an event or an ensemble of events, such as a world line, world tube, etc. Let a phenomenon with a recognisable identity (see equation (5.21)) be called an *object*. For instance, a hydrogen atom is an object.

Observation is a frame-dependent concept. An observation is, for instance, a reference event in an instrument, i.e. reference frame S, designed to measure coordinates of an arbitrary measured event O. In a given reference frame S, a reference event o defines the coordinates of a measured event O (see § 4). Instead of a single measured event there can be an ensemble of measured events which all together, if they have a recognisable identity, represent an object. To an ensemble of measured events there corresponds an ensemble of reference events, or, in other words, to a phenomenon O there corresponds an observation $o = O(S)$ performed with an instrument S. In particular, O can be an object.

For a fixed object O, *a change of reference frame*:

$$S \to S' = A^{-1} S \tag{5.1}$$

results in a change of observation:

$$o \to o' = Ao \qquad o = O(S) \qquad o' = O(S') \tag{5.2}$$

where A is a transformation of the covariance group.

In a fixed reference frame S, *a change of object*:

$$O \to O' = AO \tag{5.3}$$

results in a change of observation:

$$o \to o' = Ao \qquad o = O(S) \qquad o' = O'(S). \tag{5.4}$$

Let the *observations* o and o' which are associated with each other by a transformation A of the covariance group $\mathscr{A}$ be called *equivalent observations*, regardless of whether o and o' are due to different frames S and S', respectively, or to different objects O and O'. Equivalent observations belong to an equivalence class of observations (§ 2).

Let the *instruments* S and S' which can be mapped one into the other by $A \in \mathscr{A}$ be called *equivalent instruments* or *reference frames.* They belong to an equivalence class of instruments (§ 2).

Let the *objects* O and O' which, in a given reference frame S, can be mapped one into the other by a transformation A of the covariance group $\mathscr{A}$ be called *identical objects.*

From equations (5.1)–(5.4) and from the relation $AO(A^{-1}S) = O(S)$ it follows that

$$O(S) = O'(S'). \tag{5.5}$$

Equation (5.5) means that the observation o of the object O in the frame S is identical to the observation o'' of the object O' in the frame S'. This means that O looks in S exactly as O' does in S'.

Now, let the covariance group $\mathscr{A}$ be the inhomogeneous group ISO(4, 2), and let the transformation A be $L_{\mathrm{I}} \in \mathrm{ISO}(4, 2)$. Let S be a reference frame in which the metric tensor γ_{ab} is given by equation (3.2). Such a reference frame will be called *inertial.* Any other frame $S' = L_{\mathrm{I}}^{-1}S$ is then also inertial.

The covariance group ISO(4, 2) is inhomogeneous. It is the direct product of *the group of translations in the space V_6*:

$$\eta'^a = \eta^a + \beta^a \tag{5.6}$$

and of the homogeneous group SO(4, 2) of *the rotations* $L \in \mathrm{SO}(4, 2)$:

$$\eta'^a = L^a{}_b \eta^b. \tag{5.7}$$

Here we take a particular example of observation, namely coordinates, and a particular example of phenomenon, namely event. Instead of coordinates we can take any other observation that can be represented by components of a 6-vector. The equivalence class consists of the 6-vectors that differ among themselves up to a transformation $L_{\mathrm{I}} \in \mathrm{ISO}(4, 2)$.

Now, let us consider some particular examples of the homogeneous transformations L.

(i) The rotations of η^1, η^2, η^3 which leave the axes η^0, η^5, η^6 unchanged; they are equivalent to the usual rotations of x^1, x^2, x^3.

(ii) The rotations of η^μ, η^6 ($\mu, \nu = 0, 1, 2, 3$) which leave $\eta^5 \equiv \kappa$ unchanged:

$$\begin{aligned} \eta'^\mu &= \eta^\mu + a^\mu \kappa \\ \kappa' &= \kappa \\ \lambda' &= \lambda + 2a_\nu \eta^\nu + a^\nu a_\nu \kappa. \end{aligned} \tag{5.8}$$

In terms of the coordinates $x^a = (x^\mu, \alpha, \Lambda)$ (see equations (3.5)) this reads:

$$\begin{aligned} x'^\mu &= x^\mu + a^\mu \\ \alpha' &= \alpha \\ \Lambda' &= \Lambda. \end{aligned} \tag{5.9}$$

These are just the translations of the coordinates x^μ.

(iii) The rotations of η^5, η^6 which leave η^μ unchanged:

$$\begin{aligned} \eta'^\mu &= \eta^\mu \\ \kappa' &= \rho^{-1}\kappa \\ \lambda' &= \rho\lambda. \end{aligned} \tag{5.10}$$

In terms of the coordinates x^a this reads:

$$\begin{aligned} x'^\mu &= \rho x^\mu \\ \alpha' &= \rho\alpha \\ \Lambda' &= \rho\Lambda. \end{aligned} \tag{5.11}$$

These are just *the dilatations* of the coordinates x^a.

(iv) The rotations that leave η^0, η^6 unchanged but change η^r, η^5 $(r = 1, 2, 3)$:

$$\begin{aligned} \eta'^0 &= \eta^0 \\ \eta'^r &= \eta^r - c^r\lambda \\ \kappa' &= \kappa - 2c_r\eta^r + c^r c_r\lambda \\ \lambda' &= \lambda. \end{aligned} \tag{5.12}$$

(v) A rotation L which is *a generalisation of the usual Lorentz transformations* can be represented by the matrix:

$$L = \begin{pmatrix} \gamma & \nu_s\gamma & \nu_5\gamma & \nu_6\gamma \\ -\nu^r\gamma & 1 - A\nu^r\nu_s & -A\nu^r\nu_5 & -A\nu^r\nu_6 \\ -\nu^5\gamma & -A\nu^5\nu_s & 1 - A\nu^5\nu_5 & -A\nu^5\nu_6 \\ -\nu^6\gamma & -A\nu^6\nu_s & -A\nu^6\nu_5 & 1 - A\nu^6\nu_6 \end{pmatrix} \tag{5.13}$$

where $\nu^a \equiv \mathrm{d}\eta^a/\mathrm{d}\eta^0 \equiv (\nu^\mu, \nu^5, \nu^6)$, $\nu_a \equiv (\nu_\mu, \nu_5, \nu_6) = (\nu_\mu, -\frac{1}{2}\nu_6, -\frac{1}{2}\nu_5)$ is a velocity with respect to a frame S of a world line which is at rest in the frame S'. Let the component $\nu^5 \equiv \mathrm{d}\kappa/\mathrm{d}\eta^0$ be called the *dilatational speed.* Further notation in equation (5.13) is $\gamma = (\nu_a\nu^a)^{-1/2}$, $A = \gamma^2(1+\gamma)^{-1}$, where $\nu_a\nu^a \equiv 1 + \nu_r\nu^r + \nu_6\nu^6 = 1 - \nu^r\nu^r - \nu^5\nu^6$. The matrix L, given by equation (5.13), has the same form when expressed *both* in the coordinates η^a in which the metric tensor δ_{ab} is non-diagonal (equation (3.2)) *and* in the coordinates $\bar{\eta}^a$ in which the metric tensor is diagonal.

In the case $\nu^a = (1, \nu^r, 0, 0)$ equation (5.13) represents the usual Lorentz matrix.

In the case $\nu^a = (1, 0, \nu^5, \nu^6)$ equation (5.13), when inserted into equation (5.7), yields:

$$\begin{aligned} \eta'^0 &= \gamma(\eta^0 - \tfrac{1}{2}\nu^6\kappa - \tfrac{1}{2}\nu^5\lambda) && [\kappa \equiv \eta^5, \lambda \equiv \eta^6] \\ \eta'^r &= \eta^r \\ \kappa' &= -\nu^5\gamma\eta^0 + (1 + \tfrac{1}{2}A\nu^5\nu^6)\kappa + \tfrac{1}{2}A\nu^5\nu^5\lambda && [A = \gamma^2(1+\gamma)^{-1}] \\ \lambda' &= -\nu^6\gamma\eta^0 + \tfrac{1}{2}A\nu^6\nu^6\kappa + (1 + \tfrac{1}{2}A\nu^5\nu^6)\lambda && [\gamma = (1 - \nu^5\nu^6)^{-1/2}]. \end{aligned} \tag{5.14}$$

This is a transformation of the coordinates η^a from a frame S, in which $\nu^5 = \mathrm{d}\kappa/\mathrm{d}\eta^0$ of a world line is zero, to a frame S' in which $\nu^5 \neq 0$ and $\nu^6 \neq 0$.

In the case $\nu^a = (1, 0, \nu^5, 0)$ equation (5.14) reduces to

$$\begin{aligned} \eta'^0 &= \eta^0 - c^0\lambda \qquad (c^0 \equiv \nu^5/2) \\ \eta'^r &= \eta^r \\ \kappa' &= \kappa - 2c^0\eta^0 + (c^0)^2\lambda \\ \lambda' &= \lambda. \end{aligned} \tag{5.15}$$

If we apply first the transformation (5.12) and then the transformation (5.15), or vice versa, we obtain the following rotation in V_6:

$$\begin{aligned} \eta'^\mu &= \eta^\mu - c^\mu\lambda \\ \kappa' &= \kappa - 2c_\nu\eta^\nu + c^\nu c_{\nu\lambda} \\ \lambda' &= \lambda. \end{aligned} \tag{5.16}$$

In the presence of *the constraint* $\eta^a\eta_a = 0$ (i.e. $\lambda = \kappa x^\nu x_\nu$), equation (5.16) is equivalent to the special conformal transformations in M_4:

$$x'^\mu = (x^\mu - c^\mu x^\nu x_\nu)/\sigma(x) \qquad \sigma(x) = 1 - 2c_\nu x^\nu + c_\nu c^\nu x_\alpha x^\alpha. \tag{5.17}$$

As the constraint $\eta^a\eta_a = 0$ is not compatible with the principle of relativity we abolish such a constraint and work with six independent coordinates η^a, giving a physical meaning also to the sixth coordinate $\eta^6 \equiv \lambda$.

So far, we have used a single event. Equations (5.6) and (5.7) represent either a mapping of an event E into another event E':

$$E \xrightarrow{L_1} E' \tag{5.18}$$

in a given reference frame, or a change of coordinates of an event E when passing from a reference frame S into another frame S'. In both cases the coordinates of E change:

$$\eta^a \xrightarrow{L_1} \eta'^a.$$

Instead of a single event we can map an ensemble of events E_i (i runs over the ensemble) into another ensemble E'_i:

$$E_i \xrightarrow{L_1} E'_i \qquad (i = 1, 2, \ldots, N), \tag{5.19}$$

where N is the number of events in the ensemble. The coordinates of an ensemble of events transform according to (5.6) and (5.7) but instead of η^a we use η_i^a:

$$\eta_i^a \xrightarrow{L_1} \eta_i^a \qquad (i = 1, 2, \ldots, N). \tag{5.20}$$

An ensemble of events E_i with coordinates η_i^a can represent *an object* O:

$$O \leftrightarrow \eta_i^a \qquad (i = 1, 2, \ldots, N).$$

N is then the number of events which sample an object.

An object has been defined as a phenomenon with a recognisable identity.

In terms of the coordinates η^a, let *the identity* $\mathcal{I}$ of an object be defined as the totality of 6-distances η_{ij} between events E_i and E_j:

$$\mathcal{I} \leftrightarrow \eta_{ij}^2 = (\eta_i^a - \eta_j^a)(\eta_{ia} - \eta_{ja}) \qquad (i, j = 1, 2, \ldots, N). \tag{5.21}$$

We can define the 6-momentum by

$$\pi^a = m_{00}\, \mathrm{d}\eta^a/\mathrm{d}\sigma \tag{5.22}$$

where m_{00} is the mass (Barut and Haugen 1972) invariant under transformations of the group ISO(4, 2). Let the component π^5 be called *dilatational momentum.* A world line of a point particle is characterised by its 6-momentum π^a. The latter transforms as a 6-vector, according to equations (5.6) and (5.7), where η^a is replaced by π^a.

In a given reference frame, an ensemble of world lines is characterised by a corresponding ensemble of momenta π_i^a $(i = 1, 2, \ldots, M)$ which, under a transformation L_I ISO(4, 2), transforms according to (5.6) and (5.7):

$$\pi_i^a \xrightarrow{L_\mathrm{I}} \pi_i'^a, \qquad (i = 1, 2, \ldots, M).$$

M is the number of world lines in an ensemble.

An ensemble of world lines W_i with momenta π_i^a can also represent an object O:

$$O \leftrightarrow \pi_i^a \qquad (i = 1, 2, \ldots, M). \tag{5.23}$$

In terms of momenta π^a, let the identity $\mathscr{I}$ of an object be defined as the totality of 6-distances π_{ij} in momentum space:

$$\mathscr{I} \leftrightarrow \pi_{ij}^2 = (\pi_i^a - \pi_j^a)(\pi_{ia} - \pi_{ja}) \qquad (i, j = 1, 2, \ldots, M). \tag{5.24}$$

Instead of the coordinates η^a we can use the coordinates x^a which, according to our definition (see § 2), are directly observed in experiments. Let a^a and a_a be, respectively, contravariant and covariant components of a 6-vector, expressed in the coordinates η^a. Corresponding components A^a and A_a in the coordinates x^a are then (Fulton 1962):

$$A^a = \kappa^{-n+2} \frac{\partial x^a}{\partial \eta^b} a^b \qquad A_a = \kappa^{-n} \frac{\partial \eta^b}{\partial x^a} a_b \tag{5.25}$$

where n is *the degree of homogeneity* (see Barut and Haugen 1972). Namely, all functions in coordinates η^a are assumed to be homogeneous functions satisfying

$$a(\rho\eta) = \rho^n a(\eta). \tag{5.26}$$

Equation (5.25) implies

$$A^a A_a = \kappa^{-2n+2} a^a a_a. \tag{5.27}$$

The degree of homogeneity of the 6-momentum $\pi^a = m_{00}\, \mathrm{d}\eta^a/\mathrm{d}\sigma$ is $n = 0$. Let p^a be the 6-momentum in the coordinates x^a. It is related to π^a according to

$$\begin{aligned} p^\mu &= \kappa(\pi^\mu - x^\mu \pi^5) \\ p^5 &= -\pi^5 \\ p^6 &= \kappa^2(\pi^6 - 2x_\nu \pi^\nu + x^\nu x_\nu \pi^5) \end{aligned} \tag{5.28}$$

where we have taken into account (5.25), (3.5) and $n = 0$. Similarly, for covariant components (also in agreement with (5.25), (3.5) and $n = 0$):

$$p_\mu = \kappa(\pi_\mu - x_\mu \pi^5) \qquad p_5 = \tfrac{1}{2} p^6 \qquad p_6 = \tfrac{1}{2} p^5. \tag{5.29}$$

From (5.22) and (5.28):

$$p^a = \kappa m_{00} \frac{dx^a}{d\tilde{s}} \qquad p_a = \kappa m_{00} \frac{dx_a}{d\tilde{s}} \tag{5.30}$$

where $x^a \equiv (x^\mu, \alpha, \Lambda)$, $x_a = (x_\mu, \frac{1}{2}\Lambda, \frac{1}{2}\alpha)$, $d\tilde{s}^2 \equiv dx^\mu \, dx_\mu + d\alpha \, d\Lambda$ (see equation (3.6)). Equations (5.30) and (5.22) imply

$$p^a p_a = \kappa^2 m_{00}^2 = \kappa^2 \pi^a \pi_a \tag{5.31}$$

which is in agreement with equation (5.27).

If we use the coordinates x^a, then *the identity* $\mathcal{I}$, as defined by equation (5.21), assumes the form

$$\mathcal{I} \leftrightarrow \eta_{ij}^2 = \kappa_i \kappa_j [(x_i^\mu - x_j^\mu)(x_{i\mu} - x_{j\mu}) + (\alpha_i - \alpha_j)(\Lambda_i - \Lambda_j)]. \tag{5.21a}$$

Similarly, the identity $\mathcal{I}$, as defined by equation (5.24), can be written in the form:

$$\mathcal{I} \leftrightarrow \pi_{ij}^2 = (\alpha_i p_i^\mu - \alpha_j p_j^\mu)(\alpha_i p_{i\mu} - \alpha_j p_{j\mu}) + (p_i^5 - p_j^6)(\alpha_i^2 p_i^5 - \alpha_j^2 p_j^6) \tag{5.24a}$$

where equations (5.28) and (5.29) have been used, with $\alpha_i = \kappa_i^{-1}$.

An object O can be represented either by events E_i or world lines W_i. A transformation L_{I} maps an object O into another object O' which is represented by events E_i' or world lines W_i', and has the same identity $\mathcal{I}$ as the object O.

The identity $\mathcal{I}$ in terms of six coordinates η^a or momenta π^a is invariant under any transformation of the group ISO(4, 2):

$$\begin{aligned} \eta_{ij}'^2 &= \eta_{ij}^2 \\ \pi_{ij}'^2 &= \pi_{ij}^2. \end{aligned} \tag{5.32}$$

In terms of the usual four coordinates x^μ or momenta p^μ we can define *the Minkowski identity I*:

$$I \leftrightarrow x_{ij}^2 = (x_i^\mu - x_j^\mu)(x_{i\mu} - x_{j\mu}) \qquad (i, j = 1, 2, \ldots, N) \tag{5.33}$$

$$I \leftrightarrow p_{ij}^2 = (p_i^\mu - p_j^\mu)(p_{i\mu} - p_{j\mu}) \qquad (i, j = 1, 2, \ldots, M) \tag{5.34}$$

which, in general, is not invariant under $L_{\mathrm{I}} \in$ ISO(4, 2).

Analogously, in terms of three coordinates x^1, x^2, x^3 we can define the three-dimensional identity of objects which is an invariant concept only with respect to three-dimensional rotations. Special relativity in M_4 has replaced the three-dimensional identity by the four-dimensional Minkowski identity I. Special relativity in V_6 replaces the Minkowski identity I by the six-dimensional identity $\mathcal{I}$.

From equations (5.32), (5.21a) and (5.24a) it follows that both x_{ij} *and* p_{ij}, *defined by equations (5.33) and (5.34), are invariant under those transformations of ISO(4, 2) that leave* α, Λ, p^5, p^6 *invariant.* Such transformations are the usual translation and Lorentz transformations of x^μ and p^μ, belonging to the Poincaré group ISO(3, 1) $\subset$ ISO(4, 2). They conserve the Minkowski identity of an object. However, if the covariance group is ISO(4, 2) and not ISO(3, 1), then the Minkowski identity I is not an invariant. It must be replaced by the six-dimensional identity $\mathcal{I}$, as defined by equations (5.21) and (5.24) or (5.21a) and (5.24a), which is invariant under any transformation of the group ISO(4, 2).

Roughly speaking, transformations which go beyond the Poincaré group would, in general, change the objects' sizes and forms, as observed in terms of *four* coordinates

x^μ. Their Minkowski identities I will be, in general, transformed. But their six-dimensional identities $\mathscr{I}$ would remain invariant, as well as their invariant masses m_{00}. A prediction of the theory is that such objects should exist. A justification for such a prediction has been made in § 1 (see also § 6).

Let us consider, for instance, a *dilatation*, defined by equations (5.10). From equations (5.32), (5.21*a*) and (5.24*a*) it follows that under a dilatation the quantities x_{ij} and p_{ij} transform according to

$$x'_{ij} = \rho x_{ij} \tag{5.35}$$

$$p'_{ij} = \rho^{-1} p_{ij}. \tag{5.36}$$

If our object is a hydrogen atom, then x_{ij} represent the intrinsic distribution of the electron cloud, whilst p_{ij} represent its intrinsic configuration of energy and momentum states (instead of linear momenta p_i^a we could use angular momenta in the definition of the identity $\mathscr{I}$). A dilatation maps a hydrogen atom H with η_{ij}, π_{ij} and invariant mass m_{00} into another hydrogen atom H′ with $\eta'_{ij} = \eta_{ij}$, $\pi'_{ij} = \pi_{ij}$ and $m'_{00} = m_{00}$. The atom H has x_{ij}, p_{ij}, but the transformed atom H′ has $x'_{ij} = \rho x_{ij}$, $p'_{ij} = \rho^{-1} p_{ij}$.

There is no contradiction with quantum mechanics, since the latter must also be extended to the space V_6 (Barut and Haugen 1972, 1973, Pavšič 1977). For instance, the Dirac equation in V_6 can be written (Barut and Haugen 1973, Pavšič 1977) in the form

$$[\gamma_a(\hat{\pi}^a - e\varphi^a) - m_{00}]\psi = 0 \qquad (\hbar = c = 1) \tag{5.37}$$

where γ_a are the Dirac matrices in V_6, e is the electric charge, φ^a is the electromagnetic field potential, and $\hat{\pi}^a = -\mathrm{i}\partial/\partial\eta^a$. Let the eigenvalues of $\hat{\pi}^a$ be π_i^a. The latter eigenvalues enter the expressions (5.21) and (5.24) for the identity $\mathscr{I}$. *The Planck constant* $\hbar = 1$ *is taken to be invariant under any transformation of* $ISO(4, 2)$. *It fixes the scale of length in terms of the coordinates* η^a, *i.e. it fixes* $\mathrm{d}\sigma$ *(see equations (3.6) and (3.7)),* η_{ij} *and* π_{ij}, *but it does not fix the scale of* x^a, $\mathrm{d}s$, x_{ij} *and* p_{ij}. *Both atoms H and H′ have energy and angular momentum states which are solutions of the same Dirac equation in* V_6.

A position in V_6 of an atom or any other particle, obeying quantum mechanics in V_6, is not restricted to one particular value of scale $\eta^5 \equiv \kappa$. However, the analogy with the usual quantum mechanics suggests that, when particles interact among themselves so as to form bound systems, *their scales, due to quantum effects in* V_6, *assume certain discrete values* α_i. This is a reason why the scales of particles appear to be fixed. Consider a *crystal.* Its constituent atoms, due to quantum effects, *have fixed discrete positions* x_i^r ($r = 1, 2, 3$, i runs over all atoms in the crystal) *with respect to the crystal.* But a crystal as a whole can be continuously translated. Each individual atom, if being free—and not bound within a crystal—can assume an arbitrary position x^r. Bear in mind that in the present theory the analogy between discrete positions x_i^r and discrete scales α_i, and the analogy between free positions x^r and free scales α, is justified by the fact that both x^r and α are treated on the same footing, being merely components of the 6-vector x^a (or η^a).

It has been realised by Barut (1977) that different states of the same hydrogen atom can be mapped, by a change of scale, into each other. This is in a sense analogous to the fact that different discrete positions of atoms within a crystal can be mapped into each other by appropriate translations.

However, this is not the whole story: a whole crystal can be *continuously translated* without changing its Minkowski identity I (equations (5.33) and (5.34)), i.e. without changing relative positions of its constituent atoms. The atoms themselves retain their individual Minkowski identities.

Analogously, a whole crystal can be *continuously dilated* without changing its 6-dimensional identity $\mathscr{I}$. Its Minkowski identity is then changed by a continuous factor ρ (see equations (5.35) and (5.36)). I have already shown in § 1 that present observations do not exclude such a possibility, but rather indicate the contrary (see § 6).

One example was dilatation. Similarly we can investigate what happens to an object if transformed by any other transformation of the group ISO(4, 2). In any case, its six-dimensional identity $\mathscr{I}$ is conserved (see equation (5.32)), but its Minkowski identity I (equations (5.33) and (5.34)) is not always conserved.

Let us also consider a transformation (5.15) which implies

$$\frac{\alpha}{\alpha'}=\frac{1-2c_0t}{1-c_0^2\lambda\alpha'}. \tag{5.38}$$

From equations (5.38), (5.32) and (5.21*a*) it follows that

$$x_{ij}=\frac{\alpha}{\alpha'}x'_{ij} \tag{5.39}$$

provided that we take $t_i\equiv t_j=t$, $\alpha_i=\alpha_j\equiv\alpha$ and $\lambda_i=\lambda_j\equiv\lambda$. A consequence of this last assumption and equation (5.15) is that $\alpha'_i=\alpha'_j\equiv\alpha'$ and $\lambda'_i=\lambda'_j\equiv\lambda'=\lambda$.

Equations (5.38) and (5.39) imply that if x'_{ij} are constant at various times $t_i=t_j=t$, then x_{ij} are not constant, but change.

We say that in the frame S an object O with variable x_{ij} is *moving dilatationally* with a dilatational speed $v^5=2c_0$ (see equation (5.15)). In a frame S', related to S by the transformation (5.15), the same object O with constant x'_{ij} is said to be *dilatationally at rest*: $v'^5=0$.

In the frame S objects O and O' are predicted to exist, which are mapped into each other by the transformation (5.15). The latter object is dilatationally at rest, whilst the former one is moving dilatationally.

This implies, heretically as it may appear, that any object can start 'shrinking or expanding before our eyes', provided that it is given (for instance, in an interaction or a collision, see equation (5.42)) an appropriate *dilatational momentum* π^5. Present observations do not exclude the possibility that galaxies are moving dilatationally with respect to each other. Later we shall see that if an electron is supposed to be dilationally at rest, whilst a muon is moving dilatationally, then we have a description of their observed mass ratio (§ 6).

Let us now explain the requirements (a), (b), (c) of the principle of relativity for the example of the covariance group ISO(4, 2).

(a) *Equivalence classes of observations are isomorphic to each other.* If two different phenomena O and O' are identical, i.e. if the frames S and S' exist such that equation (5.5) is satisfied, then their observations o and o' belong to the same equivalence class. On the other hand, if O and O' are not identical, so that it is not possible to find two frames S, S' in which equation (5.5) would be satisfied, observations $o=O(S)$ and $o'=O'(S)$ belong to different equivalence classes. *Observations of different non-identical objects belong to different equivalence classes.*

An object has been defined as an ensemble of events E_i with observed coordinates η_i^a having an identity $\mathscr{I}$. Different objects O_1 and O_2 are represented by different ensembles of coordinates $\eta_i^a(1)$ and $\eta_i^a(2)$, respectively, which can be transformed according to equations (5.6) and (5.7) into $\eta_i'^a(1)$ and $\eta_i'^a(2)$, respectively. It is evident that the set $\{\eta_i^a(1), \eta_i'^a(1), \ldots\}$ is isomorphic to the set $\{\eta_i^a(2), \eta_i'^a(2), \ldots\}$, i.e. their elements are in one-to-one correspondence. For instance, identical crystals, translated in all possible ways, are in one-to-one correspondence with identical stones, also translated in all possible ways. Translations, Lorentz transformations, dilatations, etc, can be applied to all phenomena including all objects such as planets, stars, crystals, etc. All these objects have the same covariance group, in our case ISO(4, 2).

(b) *Isomorphism between the set of frames and any equivalence class of observations.* This requirement means that reference frames are in one-to-one correspondence with the observations of an equivalence class. This is true, as follows from requirement (a) only if *reference frames are identical objects.* If these identical objects are free, i.e. move freely translationally, dilatationally, etc, so that the metric is given by equation (3.2), then we say that the reference frames are *inertial.*

(c) *Law of motion.* In the present paper we restrict ourselves to *the flat space* V_6, even though an extension to a curved V_6 has been already initiated in Pavšič (1977).

A *free object* moves from a point η_1^a to a point η_2^a so that the path is extremal:

$$\delta \int_{\eta_1^a}^{\eta_2^a} \mathrm{d}\sigma = 0. \tag{5.40}$$

The differential equation, following the variational principle from (5.40), is

$$\mathrm{d}^2\eta^a/\mathrm{d}\sigma^2 = 0. \tag{5.41}$$

If the object is *not free*, then we can assume that it is under the influence of a 6-force f^a, so that the law of motion is

$$\mathrm{d}\pi^a/\mathrm{d}\sigma = f^a. \tag{5.42}$$

The quantity $\pi^a \equiv m_{00}\,\mathrm{d}\eta^a/\mathrm{d}\sigma$ is the 6-momentum, m_{00} being the invariant mass (Barut and Haugen 1972) with respect to ISO(4, 2). The momentum π^a of an isolated object is a constant of motion. If free particles collide, their total momentum π^a is conserved. If initially a particle O_1 has the dilatational momentum $\pi_1^5 = 0$ and collides with another particle O_2 having $\pi_2^5 \neq 0$, then after the collision O_1 has $\pi'^5_1 \neq 0$, i.e. it starts moving dilatationally.

In terms of the coordinates x^a (see equation (3.5)) the law of motion (5.42) reads:

$$\frac{\mathrm{d}p^a}{\mathrm{d}\tilde{s}} = \tilde{F}^a \qquad \mathrm{d}\tilde{s} \equiv (\mathrm{d}x^a\,\mathrm{d}x_a)^{1/2} \tag{5.43}$$

where $p^a = \kappa m_{00}\,\mathrm{d}x^a/\mathrm{d}\tilde{s}$ (see also equation (5.28)). If we insert into equation (5.43) the expressions:

$$p^\mu = m_0^*\,\mathrm{d}x^\mu/\mathrm{d}s \qquad m_0^* = \kappa m_{00}\,\mathrm{d}s/\mathrm{d}\tilde{s} \qquad F^\mu = \tilde{F}^\mu\,\mathrm{d}\tilde{s}/\mathrm{d}s$$
$$\mathrm{d}s \equiv (\mathrm{d}x^\mu\,\mathrm{d}x_\mu)^{1/2}$$

and if we restrict ourselves only to the first four components of equation (5.43), then we obtain

$$\mathrm{d}p^\mu/\mathrm{d}s = F^\mu \tag{5.44}$$

which is the usual law of motion, but with *the effective mass* m_0^* which can be written also as $m_0^* = (p^\mu p_\mu)^{1/2}$. In the case of free motion, i.e. for $f^\mu \neq 0, f^5 = f^6 = 0$ and therefore π^5 and π^6 being constant, we have

$$m_0^* = \kappa(0)(m_{00}^2 + \pi^5\pi^6(0))^{1/2} \tag{5.45}$$

where $\pi^6(0)$ and $\kappa(0)$ are the values of π^6 and κ taken at $x^\mu = 0$, respectively. The effective mass m_0^* is a *Poincaré-invariant.* It is also a constant of motion, provided that $f^5 = f^6 = 0$. When studying translational motion of the centre of mass of a particle, we cannot distinguish between the case $\pi^5 = 0$ and $\pi^5 \neq 0$, except for the differences in the effective masses. The influence of non-zero dilatational momentum $\pi^5 \neq 0$ is thus hidden. This is a reason why we have not observed dilatational motions of subnuclear particles, even if such motions indeed exist.

6. Some further physical implications

6.1. Anomalous red shifts of galaxies. In the last few years pairs of galaxies have been observed connected by a 'bridge' and which have red shifts different up to a factor of 2 (Arp 1970). This has been a cause of the controversy (Field *et al* 1973) whether the two galaxies in a pair are both at the same distance from us, or not. If they are, then their red shifts are anomalous, not obeying Hubble's law. The differences in red shifts are too great to be caused either by relative motions or by gravitational fields. Here I propose an explanation: the galaxies in a pair have different scales, and therefore, according to equation (1.3) or (1.5), different red shifts. The blue shifts are masked by the overall expansion of the universe.

6.2. Large 'velocity' dispersion in clusters of galaxies. The very large 'velocity' dispersion in clusters of galaxies has been a major puzzle in astronomy for many years (Clube 1978). The galaxies in many clusters seem to have such large random velocities—observed as random lineshifts—that the clusters can only remain gravitationally bound if some 'extra' mass, over and above that in the visible stars is present. Such 'missing' mass has not been observed. The scale degree of freedom suggests an explanation: galaxies in clusters have large random scales and therefore large random red shifts (blue shifts being masked by the overall expansion of the universe). Nevertheless they remain gravitationally bound, obeying the dynamics given by equation (5.42).

6.3. Large transverse momentum phenomena. This is now a large branch of the physics of subnuclear particles. For instance, μ pairs produced in hadronic collisions have large transverse momenta p^r with respect to incident beams. Harada *et al* (1978) realised that even if all known effects are included in the calculations, there still remains unexplained a great portion of the transverse momentum. It was named 'primordial transverse momentum of partons'.

From the relativity in V_6 it follows that each translationally *and* dilatationally moving object has necessarily a transverse momentum with respect to the direction of its translational motion. This comes intuitively from the fact the object is expanding or contracting. Formally this follows from equation (5.28). Let W_i be the world lines representing the object (see § 5), and let x_i^r be the coordinates of an event E_i, taken at the time t, on a world line W_i. Each world line W_i belonging to the object O has the momentum p_i^r (see equation (5.28)):

$$p_i^r = \kappa(\pi^r - x_i^r \pi^5) \qquad (r = 1, 2, 3)$$

where we have assumed that all world lines W_i have the same π^r, π^5 and κ. Let us choose a reference frame such that

$$\pi^r = (0, 0, \pi^3).$$

Then:

$$p_i^r = \kappa(-x_i^1 \pi^5, -x_i^2 \pi^5, \pi^3 - x_i^3 \pi^5). \tag{6.1}$$

From equation (6.1) it is obvious that p_i^r has *the transverse components* $-\kappa x_i^1 \pi^5$ and $-\kappa x_i^2 \pi^5$. The world line characterised by $x_i^r = 0$ has zero transverse momentum in the chosen reference frame.

If partons, and in some cases also hadrons, do move dilatationally, then they have necessarily 'primordial' transverse momenta given by equation (6.1).

6.4. Electron/muon mass ratio. Equation (5.45) can explain why the electron and muon have different observed masses. Let us assign to the electron e:

$$\pi_e^5 = 0 \tag{6.2}$$

and to the muon μ:

$$\pi_\mu^5 \neq 0$$

and let both e and μ have $\pi_e^6 = \pi_\mu^6 \neq 0$. In a previous paper (Pavšič 1977), π^6 has been given the physical interpretation of electric charge. Equations (5.45) and (6.2) give for the observed μ/e mass ratio the following expression:

$$\frac{m_\mu}{m_e} = \frac{m_0^*(\mu)}{m_0^*(e)} = \left(1 + \frac{\pi^5 \pi^6(0)}{m_{00}^2}\right)^{1/2}. \tag{6.3}$$

The numerical value of this ratio is in agreement with the measured value if π^5, $\pi^6(0)$ and m_{00} are quantised so that

$$\pi^5 \pi^6 / m_{00}^2 = (\tfrac{3}{2} \times 137)^2. \tag{6.4}$$

So we have reduced the problem of the μ/e mass ratio to the problem of calculating (6.4) (see, for instance, Barut 1978). However, now we have some understanding of the nature of the difference between electron and muon. *The electron and muon are supposed to be identical objects with different dilatational momenta π_e^5 and π_μ^5. Lepton number conservation is then nothing but conservation of the electron and muon dilatational momenta π_e^5 and π_μ^5, respectively.* The electron neutrino ν_e must then have $\pi_e^5 = 0$, whilst the muon neutrino ν_μ must have $\pi_\mu^5 \neq 0$.

The idea that the states associated with e and μ have the same 'conformally invariant mass' m_{00} but different rest masses when observed in the Minkowski space has been put forward by Barut and Haugen (1973). Their main idea is essentially the same as the present one. The difference is in the fact that they use the five-dimensional hypercone V_5, defined by $\eta^a \eta_a = 0$, embedded in the space V_6, and not the whole space V_6 as we use it. Different observed electron and muon masses come in their theory from the coupling between spin and the curvature of the space V_5. The expectation value of this coupling is zero for the electron, but different from zero for the muon. On the contrary, in the present theory I can describe the observed $e-\mu$ mass difference even without referring to spin. The electron and muon are identical particles, following different trajectories in V_6. When projected into the Minkowski space M_4 their trajectories appear to have different effective masses.

7. Conclusion

In the present paper I have introduced the special relativity in the six-dimensional space V_6. The theory is based on the *principle of relativity* which is extended so that it incorporates also *the dilatational degree of freedom*—scale. There are several already existing theories dealing with scale, but they (i) either do not interpret the scale as the dilatational degree of freedom; or (ii) are not based on the principle of relativity. It is just (i) and (ii) that I have assumed. So scale is treated on the same footing as the position or orientation of an object. Though the theory appears to be speculative at this stage, it is no less speculative to claim that scales of identical objects, such as hydrogen atoms, are always the same and constant over all the universe.

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